



# Probability and Random Processes

## ECS 315

**Asst. Prof. Dr. Prapun Sukksompong**

(ผศ.ดร.ประพันธ์ สุขสมปอง)

[prapun@siit.tu.ac.th](mailto:prapun@siit.tu.ac.th)

**Introduction**

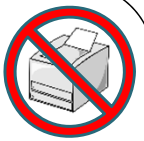


**Office Hours:**

Check Google Calendar on the course website.

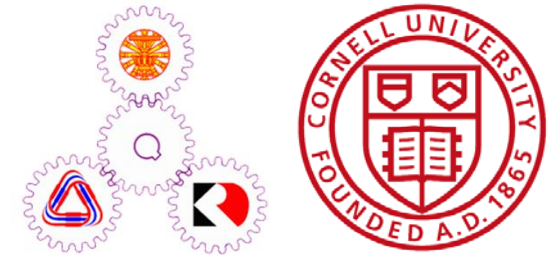
**Dr.Prapun's Office:**

6th floor of Sirindhralai building,  
BKD

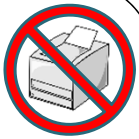


# Asst.Prof.Dr.Prapun Suksompong

- Chairperson of **Electrical Engineering Program** (and Chairperson of **Electronics and Communication Engineering** Curriculum and Electrical Engineering Curriculum) at **Sirindhorn International Institute of Technology (SIIT)**
- Ph.D. from **Cornell** University, USA
  - In Electrical and Computer Engineering
  - Minor: Mathematics (Probability Theory)
  - Research: Neuro-Information Theory (Communications in Human Brain)
- Current Research: Wireless Communications, Localization, Game Theory
- 2009, 2013, and 2017 SIIT Best Teaching Awards
- 2011 SIIT Research Award
- 2013 TU Outstanding Young Researcher Award
- 2017 SIIT Distinguished Teacher Award
- 2018 TU Outstanding Teacher in Science and Technology



[prapun.com](http://prapun.com)



# Course Syllabus



Sirindhorn International Institute of Technology  
Thammasat University

School of Information, Computer and Communication Technology

## ECS315: Course Syllabus

Semester/Year: 1/2019

**Course Title:** Probability and Random Processes  
**Instructor:** Asst. Prof. Dr.Prapun Suksompong  
([prapun@siit.tu.ac.th](mailto:prapun@siit.tu.ac.th))  
**Course Website:** <http://www2.siit.tu.ac.th/prapun/ecs315/>  
**Line Group:** <http://line.me/ti/g/V726GR67y6>



### Lectures

- Tuesday 10:40-12:00 BKD 3510
- Thursday 10:40-12:00 BKD 3506
- Friday 13:00-14:20 BKD 3511  
(Tutorial/Make-up; Shared with ECS332)

### Office Hours

See the Calendar section on the course website.

### Course Information

**Prerequisite:** MAS117 (Mathematics II: Multivariable Calculus)

**Course Description:** This course introduces the principles of probability and random processes to undergraduate students in electronics and communication. The topics to be covered include random experiments, events, probability, discrete and continuous random variables, probability density function, cumulative distribution function, functions of random variables, expectations, law of large numbers, central limit theorem, introduction to random processes, Gaussian random process, autocorrelation and power spectral density.

**Grading Policy:** Coursework will be weighted as follows:

Assignments (HWs)	5%
Class Discussion	5%
In-Class Exercises	10%
Midterm Examination	35%
Final Examination (comprehensive)	45%

- Late assignments will be heavily penalized or rejected.
- Cheating will not be tolerated

**Textbook:** [Y&G] R. D. Yates and D. J. Goodman, Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers, 2nd ed., Wiley, 2004. Call No. QA273 Y384 2005.

### Additional References:

- John A. Gubner. Probability and Random Processes for Electrical and Computer Engineers. Cambridge University Press, 2006.
- Terrence L. Fine. Probability and Probabilistic Reasoning for Electrical Engineering. Prentice Hall, 2005. Call No. QA273 F477 2005
- Henk Tijms. Understanding Probability: Chance Rules in Everyday Life. Cambridge University Press, 3rd edition, 2012. Call No. QA273 T48 2012
- William Feller. An Introduction to Probability Theory and Its Applications, Volume 1. Wiley, 3 edition, 1968.
- Probability and random processes for electrical engineering / Alberto Leon-Garcia. Call No. TK153 L425 1994
- A first course in probability / Sheldon Ross. Call No. QA273 R83 2002
- Probability models, introduction to / Sheldon M. Ross. Call No. QA273 R84 1997
- Leonard Mlodinow. The Drunkard's Walk: How Randomness Rules Our Lives. Pantheon; 8th Printing edition, 2008.

**Assignments:** Homework will be assigned throughout the semester. Most assignments will be graded on completeness, not correctness; if an honest attempt was made on an assigned problem, it will be considered complete. Occasionally, part(s) of a selected problem will be graded. Of course, you do not know which problem of which assignment will be selected; so you should work on all of them. The lowest assignment score will be dropped. The complete solutions to all problems (not just answers) will be posted on the course web site.

**Class Discussion:** The score for this part is judged by the amount of active participation in the class discussion (with the instructor) either inside or outside of the classroom. There will be (self-evaluation) forms for collecting information about this twice (one right after the midterm exam and another one right after the final exam).

**In-Class Exercises:** In-class exercises will focus on current or recently-discussed topics. An exercise may be given at any time during any class period. Students are expected to work in groups of at most three persons. In-class exercises will be given only to those students who are present. There will be no make-up exercise.

Two lowest in-class exercise scores will be dropped. Additionally, one who has legitimate excuse (such as participating in competition, or university-approved curricular and extracurricular activity, career-related interview, broken bone(s), being admitted to the hospital) may request that the corresponding missing score will not be counted. For such request, supporting document should be submitted to the instructor and the student must explicitly provide the missing exercise number and date in the (self-evaluation) forms.

**Exams:** A handwritten A4 study sheet is allowed. One side for the midterm exam. Another side for the final exam.

**Students should notify the instructor before missing any exam if at all possible and immediately thereafter when not possible.** The instructor (and/or the fact-finding committee) will determine if the absence from an exam is legitimate. Simply not feeling well is not a reason to miss an exam. In the case of legitimate absence, an oral and/or written make-up exam could be arranged.

**Expectations:** You should expect to spend extra 5-8 hours per week studying outside of class. However, the instructor *do* expect you to come to class and participate actively in class discussions. If you must miss a class, you must find out and catch up with what happened in lecture, either from the instructor or one of your classmates. You are responsible for all materials that are discussed in class.

### Academic Integrity

The work submitted in this class is expected to be the result of your individual effort. You are free to discuss course material, approaches to problems with your colleagues or the instructor but you should never misrepresent someone else's work as your own.

It is your responsibility to protect your work from unauthorized access. For example, do not discard copies of your codes/assignments in public places.

### Course Outline

The following is a tentative list of topics with their corresponding chapters from the textbook by Yates and Goodman. Each topic spans approximately one week.

- Introduction, Set Theory, Classical Probability [1]
- Combinatorics: Four Principles and Four Kinds of Counting Problems [1]
- Probability Foundations [1]
- Event-based Conditional Probability [1]
- Event-based Independence [1]
- Random variables, Support, Probability Distribution [2]
- MIDTERM: 3 Oct 2019 TIME 15:00 - 17:00**
- Discrete Random Variables [2]
- Families of Discrete Random Variables and Introduction to Poisson Processes [2,10]
- Real-Valued Functions of a Random Variable [2]
- Expectation, Moment, Variance, Standard Deviation [2]
- Continuous Random Variables [3]
- Families of Continuous Random Variables and Introduction to Poisson Processes [3,10]
- Multiple Random Variables [4-6]
- Correlation, Covariance, Limiting Theorems [4, 6, 7]
- Mixed Random Variables, Introduction to Random Vectors and Random processes [3, 5, 10]
- FINAL: 9 Dec 2019 TIME 09:00 - 12:00**

### Additional Remarks

- Calculator: Casio FX-991 is permitted in exams and for in-class exercises
- MATLAB: Computer simulation will be used to enhance learning. MATLAB is available in SIIT computer labs.

# Course Website

prapun.com



Asst. Prof. Dr. Prapun Suksomchai is an Associate Professor at Sirindhorn International University. He topped the list of

Right after his graduation, he was ranked among the top 10 other faculty members in the category of (ประเภทอาจารย์) from Thailand.

Ajarn Prapun always highly values the Student Branch "for exemplary teaching" award from Sirindhorn International University. He has received the "Distinguished Teacher" award from Sirindhorn International University.

For more information, [here is his CV](#). (CV)

## Teaching

Current version



- For 1/2019, he teaches
  - [ECS315 \(Probability and Random Processes\)](#)
  - [ECS332 \(Principles of Communications\)](#)
- For 3/2018, he taught
  - [ICT Elementary for Embedded Systems](#) (Fourier transform and principles)
- For 2/2018, he taught
  - [ECS452 \(Digital Communication Systems\)](#)

Earlier version



- For 1/2018, he taught
  - [ECS315 \(Probability and Random Processes\)](#)
  - [ECS332 \(Principles of Communications\)](#)

## ECS315: Probability and Random Processes

Everything we do, everything that happens around us, obeys the laws of probability. We can no more escape them than we can escape gravity... "Probability," a philosopher (Bishop Butler) once said, "is the very guide of life." We are all gamblers who go through life making countless bets on the outcome of countless actions.

Every field of science is concerned with estimating probability. A physicist calculates the probable path of a particle. A geneticist calculates the chances that a couple will have blue-eyed children. Insurance companies, businessmen, stockbrokers, sociologists, politicians, military experts – all have to be skilled in calculating the probability of the events with which they are concerned.

[Gardner, 1986]

### Synopsis

Probability theory is the branch of mathematics that tells us how to estimate degrees of probability. If an event is certain to happen, it is given a probability of 1. If it is certain not to happen, it has a probability of 0.

This course introduces the principles of probability and random processes to undergraduate students in electronics and communication. The topics to be covered include random experiments, events, probability, discrete and continuous random variables, probability density function, cumulative distribution function, functions of random variables, expectations, law of large numbers, central limit theorem, introduction to random processes, Gaussian random process, autocorrelation and power spectral density.

### Announcements

- Note that we also share the tutorial/make-up session with ECS332. See [Google calendar](#) below.

• This site can be accessed via [eecs315.prapun.com](#).

# Course Web Site

- Announcements
- References
- Handouts (Posted before corresponding lectures; also available at the copy center)
- Annotated Notes/Slides (Posted after corresponding lectures)
- Calendar
  - Exams
  - HW due dates



Please check the course website regularly.

[www2.siiit.tu.ac.th/prapun/ecs315/](http://www2.siiit.tu.ac.th/prapun/ecs315/)

**ECS315: Probability and Random Processes**

Everything we do, everything that happens around us, obeys the laws of probability. We can no more escape them than we can escape gravity... "Probability," a philosopher (Bishop Butler) once said, "is the very guide of life." We are all gamblers who go through life making countless bets on the outcome of countless actions.

Every field of science is concerned with estimating probability. A physicist calculates the probable path of a particle. A geneticist calculates the chances that a couple will have blue-eyed children. Insurance companies, businessmen, stockbrokers, sociologists, politicians, military experts - all have to be skilled in calculating the probability of the events with which they are concerned.

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**Announcements**

- Note that we also share the tutorial/make-up session with ECS332. See [Google calendar](#) below.
- This site can be accessed via [ecs315.prapun.com](http://ecs315.prapun.com).
- Welcome to ECS315! Feel free to look around this site.

**General Information**

- **Instructor:** Asst. Prof. Dr.Prapun Suksompong ([prapun@siit.tu.ac.th](mailto:prapun@siit.tu.ac.th))
- **Office:** BKD, 6th floor of Sirindhornalai Building
- **Office Hours:** See [Google calendar](#) below.
- **Lectures:** See [Google calendar](#) below.

**Problem Set**

a. HW 1 (Due : )

b.

**Calendar**

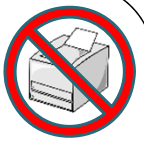
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Events shown in time zone: Indochina Time - Bangkok

**Reading Assignment**

a. Section 1.2 in the lecture notes

b. Section 2.5 in the lecture notes



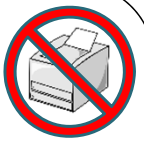
# Course Web Site

Announcements

- The **syllabus** contains tentative information.
- I will announce **in class** and on the **website** if there is any change.
- You are **responsible** for making sure that you obtain this information.
- Come to classes **on time** and listen carefully for **announcement(s)**.
- For those who want a **preview** of the class materials, old slides along with the notes and HW from **earlier years** are available on my web site (**prapun.com**).

## Teaching

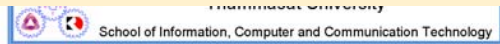
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- For 2/2017, he taugh
  - ECS452 (Dig
- For 1/2017, he taugh
  - ECS315 (Pro
  - ECS332 (Prin



# Course Website: Notes & Slides

Some **PDF notes/slides** will be posted *before* the corresponding lectures. Hard copies can also be purchased from the **copy center**.

In lecture, PDF notes/slides will be highlighted and annotated with examples / comments.



ECS315 2019/1 Part I.1 Dr.Prapun

## 1 Probability and You

Whether you like it or not, probabilities rule your life. If you have ever tried to make a living as a gambler, you are painfully aware of this, but even those of us with more mundane life stories are constantly affected by these little numbers.

**Example 1.1.** Some examples from daily life where probability calculations are involved are the determination of insurance premiums, the introduction of new medications on the market, opinion polls, weather forecasts, and DNA evidence in courts. Probabilities also rule who you are. Did daddy pass you the X or the Y chromosome? Did you inherit grandma's big nose?

Meanwhile, in everyday life, many of us use probabilities in our language and say things like "I'm 99% certain" or "There is a one-in-a-million chance" or, when something unusual happens, ask the rhetorical question "What are the odds?". [17, p 1]

### 1.1 Randomness

1.2. Many clever people have thought about and debated what randomness really is, and we could get into a long philosophical discussion that could fill up a whole book. Let's not. The French mathematician Laplace (1749-1827) put it nicely:

"Probability is composed partly of our ignorance, partly of our knowledge."



**Example 6.23.** Suppose  $\Omega = \{a, b, c, d, e\}$ . Define four events

$A = \{a, b, c\}$ ,  $B = \{a, b\}$ ,  $C = \{c, d\}$ , and  $D = \{e\}$ .

Let

$P(\{a\}) = P(\{b\}) = 0.2$ , and  $P(\{c\}) = P(\{d\}) = 0.1$ .

Calculate the following probabilities:

CH5

$$\left\{ \begin{array}{l} \text{(a) } P(\{e\}) = P(\Omega \setminus \{a, b, c, d\}) = P(\Omega) - P(\{a, b, c, d\}) \\ \quad = 1 - (0.2 + 0.2 + 0.1 + 0.1) = 1 - 0.6 = 0.4 \\ \text{(b) } P(B) = P(\{a, b\}) \quad , \quad P(C) = P(\{c, d\}) \\ \quad = 0.2 + 0.2 = 0.4 \quad \quad \quad = 0.1 + 0.1 = 0.2 \\ \quad P(D) = P(\{e\}) = 0.4 \end{array} \right.$$

$$\begin{aligned} \text{(c) } P(A|B) &\equiv \frac{P(A \cap B)}{P(B)} = \frac{P(\{a, b\})}{P(\{a, b\})} = 1 \\ P(A|C) &\equiv \frac{P(A \cap C)}{P(C)} = \frac{P(\{c\})}{P(\{c, d\})} = \frac{0.1}{0.1 + 0.1} = \frac{1}{2} \\ P(A|D) &\equiv \frac{P(A \cap D)}{P(D)} = \frac{P(\emptyset)}{0.4} = \frac{0}{0.4} = 0 \end{aligned}$$

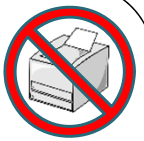
(d)  $P(A)$

Check: Observe that the collection  $\{B, C, D\}$  partitions  $\Omega$ . Use the total probability theorem to find  $P(A)$ .

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap C) + P(A \cap D) \\ &= P(A|B)P(B) + P(A|C)P(C) + P(A|D)P(D) \\ &= 1 \times 0.4 + \frac{1}{2} \times 0.2 + 0 \times 0.4 = 0.5 \end{aligned}$$

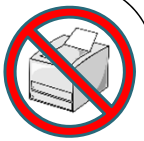
$$\text{(e) } P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)P(B)}{P(A)} = \frac{1 \times 0.4}{0.5} = \frac{4}{5} = 0.8$$

The slides and annotated notes will be **posted after** the corresponding lectures.



# Course Website: Notes & Slides

- Some **PDF notes/slides** will be posted *before* the corresponding lectures.
  - Hard copies can also be purchased from the **copy center**.
- In lectures...
  - PDF notes/slides will be highlighted and annotated with examples / comments.
    - **Put all of your energy into understanding the material.**
  - The slides and annotated notes will be **posted after** the corresponding lectures.
- **Remind** (email) me the day after the lecture if the annotated notes/slides from the day before are still not posted on the web.



# Course Organization

- **Course Website:**

<http://www2.siit.tu.ac.th/prapun/ecs315/>

- **Lectures:**

- Tuesday 10:40-12:00 BKD 3510
- Thursday 10:40-12:00 BKD 3506

- Tutorial/Exercise/Make-up sessions:

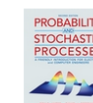
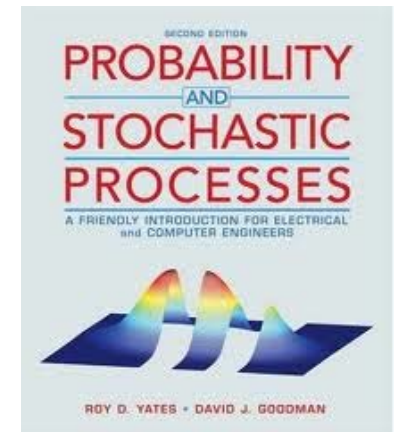
- **Friday 13:00-14:20 BKD 3511 (Shared with ECS332)**

- **Textbook:**

- Probability and stochastic processes: a friendly introduction for electrical and computer engineers

- By Roy D. Yates and David J. Goodman
- 2nd Edition
- ISBN 978-0-471-27214-4
- Library Call No. QA273Y384 2005
- Student Companion Site:

<http://bcs.wiley.com/he-bcs/Books?action=index&itemId=0471272140&bcsId=1991>



**Student  
Companion Site**

Yates, Goodman: *Probability and Stochastic Processes*:

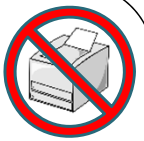
**Welcome**

Welcome to the Web site for *Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers* by Roy D. Yates and David J. Goodman. This Web site gives you access to the rich tools and resources available for this book.

1. Using the drop-down menu at the top left, select a chapter. A list of resources will be displayed.
2. Click the name of the resource you wish to use in the left navigation menu or going directly to a specific chapter.

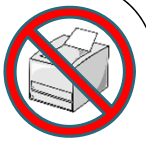
# The Friday Sessions

- Shared with ECS332.
- The first 4-5 sessions will be used for ECS315 **tutorial/review** classes.
  - Start from **this** Friday.
- Later, we will start using them as tutorial sessions.
- They can also be used for pre-announced make-up classes and in-class exercises as well.



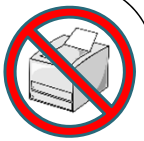
# ECS 315: Course Outline

1. Introduction, Set Theory, Classical Probability [1]
2. Combinatorics: Four Principles and Four Kinds of Counting Problems [1]
3. Probability Foundations [1]
4. Event-based Conditional Probability [1]
5. Event-based Independence [1]
6. Random variables, Support, Probability Distribution [2]
7. **MIDTERM: 3 Oct 2019 TIME 15:00 - 17:00**
8. Discrete Random Variables [2]
9. Families of Discrete Random Variables and Introduction to Poisson Processes [2,10]
10. Real-Valued Functions of a Random Variable [2]
11. Expectation, Moment, Variance, Standard Deviation [2]
12. Continuous Random Variables [3]
13. Families of Continuous Random Variables and Introduction to Poisson Processes [3,10]
14. Multiple Random Variables [4-6]
15. Correlation, Covariance, Limiting Theorems [4, 6, 7]
16. Mixed Random Variables, Introduction to Random Vectors and Random processes [3, 5, 10]
17. **FINAL: 9 Dec 2019 TIME 09:00 - 12:00**



# Probability

and Randomness



“Les questions les plus importantes de la vie ne sont en effet, pour la plupart, que des problèmes de probabilité.”

“The **most important questions** of life are, for the most part, really only problems of **probability**.”

THÉORIE  
ANALYTIQUE  
DES PROBABILITES;

PAR M. LE COMTE LAPLACE,

Chancelier du Sénat-Correspondant, Grand-Officier de la Légion d'Honneur;  
Membre de l'Institut impérial et du Bureau des Longitudes de France;  
des Sociétés royales de Londres et de Göttingue, des Académies des  
Sciences de Bologne, de Dantzick, de Suède, de Prusse, de Hollande,  
d'Italie, etc.

PARIS,

M<sup>e</sup> V<sup>e</sup> COURCIER, Imprimeur-Libraire pour les Mathématiques,  
qui de la Harpe, n<sup>o</sup> 57.

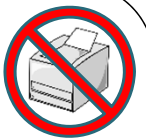
1812.

Pierre Simon Laplace (1749 - 1827)

[[https://en.wikipedia.org/wiki/Pierre-Simon\\_Laplace](https://en.wikipedia.org/wiki/Pierre-Simon_Laplace)]

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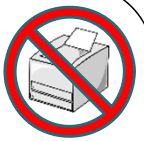
“On voit, par cet Essai, que la théorie des probabilités n'est, au fond, que le bon sens réduit au calcul; elle fait apprécier avec exactitude ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte.”

“One sees, from this Essay, that **the theory of probabilities** is basically just **common sense** reduced to **calculus**; it enables us to appreciate with exactness that which accurate minds feel with a sort of instinct, often without being able to account for it.”

THÉORIE  
ANALYTIQUE  
DES PROBABILITES;  
PAR M. LE COMTE LAPLACE,  
Chancelier du Sénat-Conservateur, Grand-Officier de la Légion d'Honneur;  
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Pierre Simon Laplace (1749 - 1827)

PARIS,  
M<sup>e</sup> V<sup>e</sup> COURCIER, Imprimeur-Libraire pour les Mathématiques,  
qui de Angoulême, n<sup>o</sup> 57.  
1812.



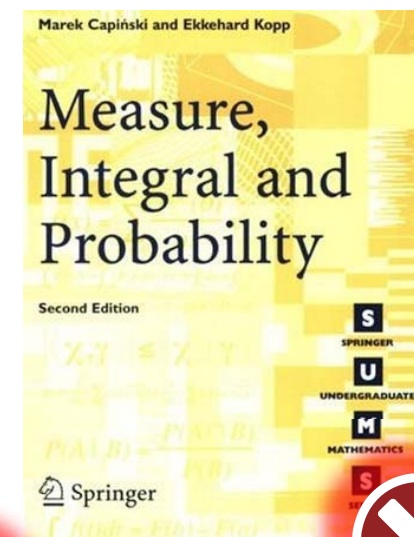
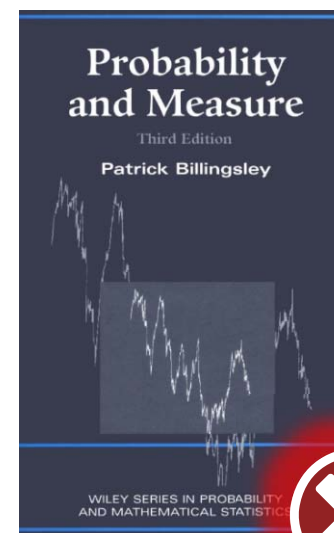
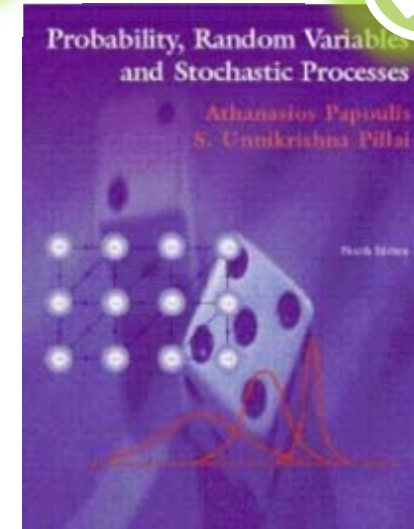
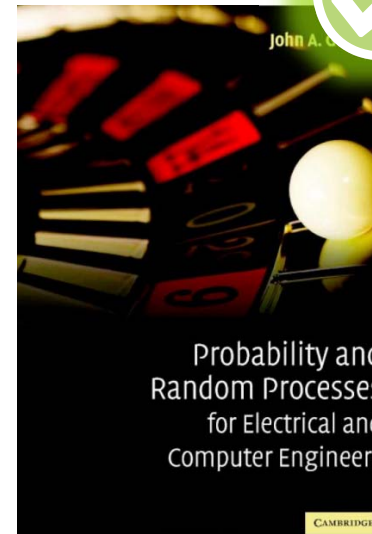
# Levels of Study in Probability Theory

- **Probability theory** is the branch of mathematics devoted to analyzing problems of chance.
  - Art of Guessing
- 1. High School: classical
- 2. Undergraduate: calculus ←———— We are here!
- 3. Graduate: measure-theoretic



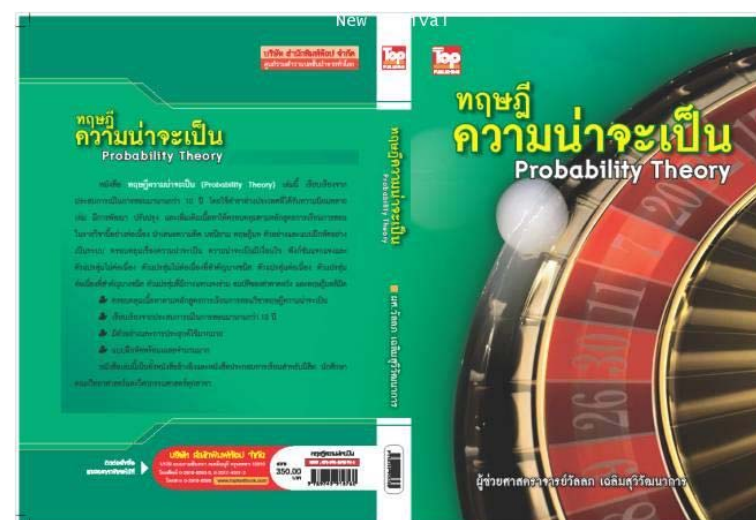
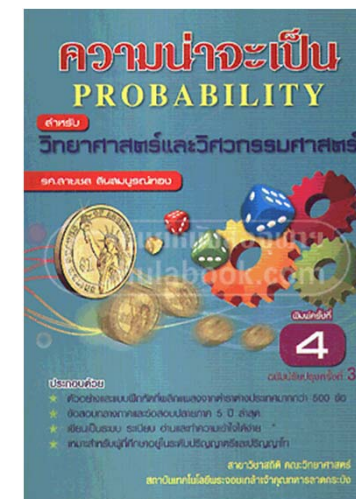
# More references

- Use ones that say probability and random (or stochastic) processes
- If it has the word “statistics” in the title, it may not be rigorous enough for this class
  - Many chapters will overlap our class content. In which case, it provide a nice reading with beautiful/colorful figures.
- If it has the word “measure” or “ergodic” in there, it is probably too advanced.



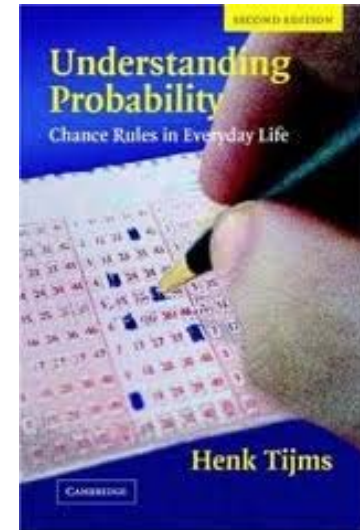
# More References (in Thai)

- ความน่าจะเป็นและสถิติสำหรับวิศวกรรมไฟฟ้า
  - ผู้แต่ง: มานพ วงศ์สายสุวรรณ และคณะ
  - ISBN : 9789740324164
- ความน่าจะเป็น :สำหรับวิทยาศาสตร์และวิศวกรรมศาสตร์ (PROBABILITY)
  - ผู้แต่ง : สายชล สิ้นสมบูรณ์ทอง
  - ISBN : 9789740329053
- ทฤษฎีความน่าจะเป็น - Probability Theory
  - ผู้เขียน: ผู้ช่วยศาสตราจารย์วัลลภ เฉลิมสุวิวัฒนาการ
  - ISBN 9789749918760



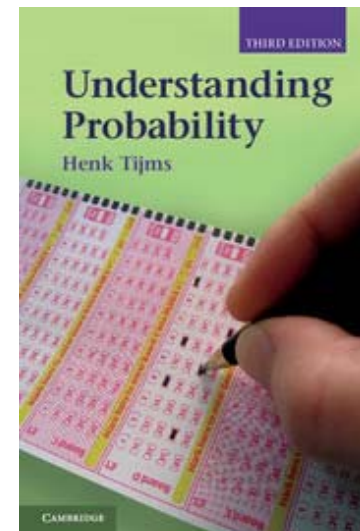
# Recommended Reading

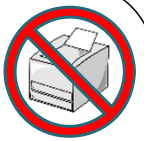
- Understanding Probability: Chance Rules in Everyday Life
- By Henk Tijms
- Call No. QA273 T48 2012
- Cambridge University Press
- “Part One” provides many motivating examples and problems from everyday life
- “Part Two” teaches clearly and simply the mathematics of probability theory.
- Sample materials are available at the author’s website: <http://personal.vu.nl/h.c.tijms/>
- <http://www.cambridge.org/aus/catalogue/catalogue.asp?isbn=9781107658561&ss=exc>



2nd Edition (2007)

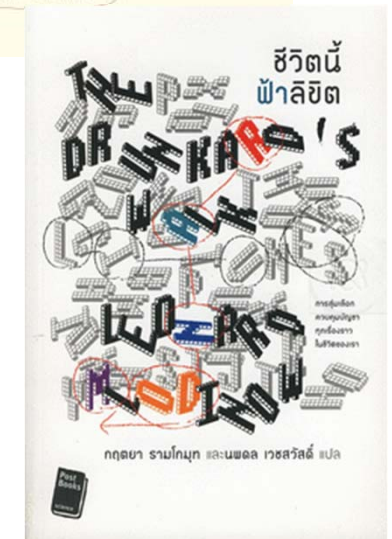
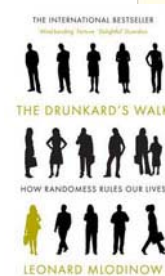
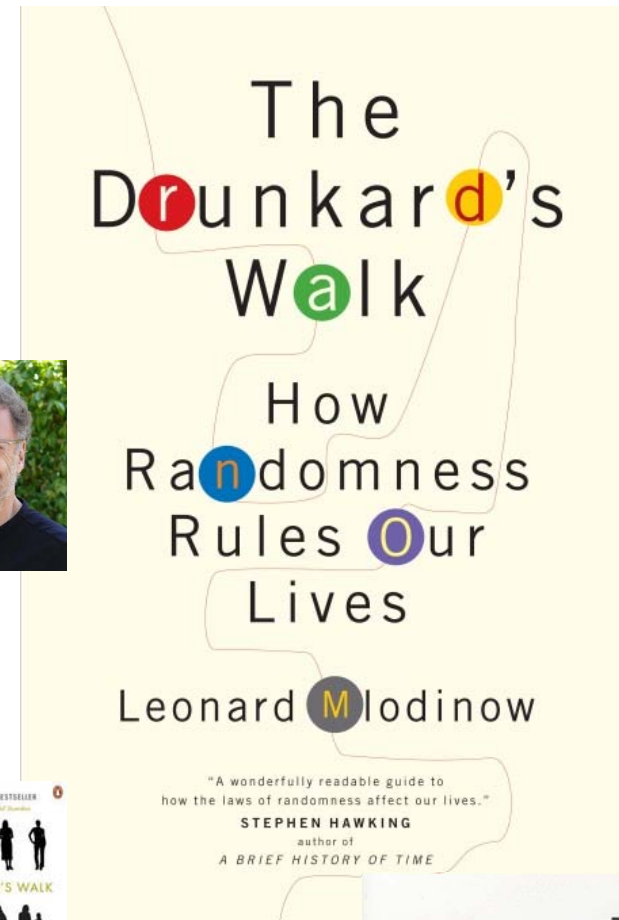
3rd Edition (2012)





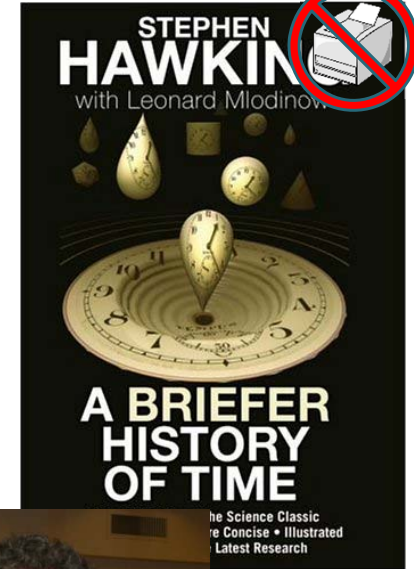
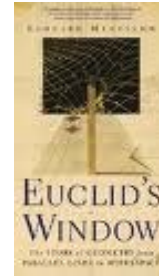
# The Drunkard's Walk

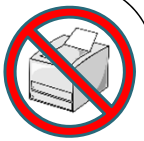
- The Drunkard's Walk: How Randomness Rules Our Lives
- By Leonard Mlodinow
- Deals with randomness and people's inability to take it into account in their daily lives.
- A bestseller, and a “NY Times notable book of the year”
- Named “one of the 10 best science books of 2008” on Amazon.com.



# Leonard Mlodinow

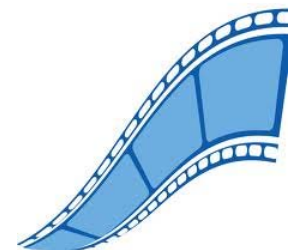
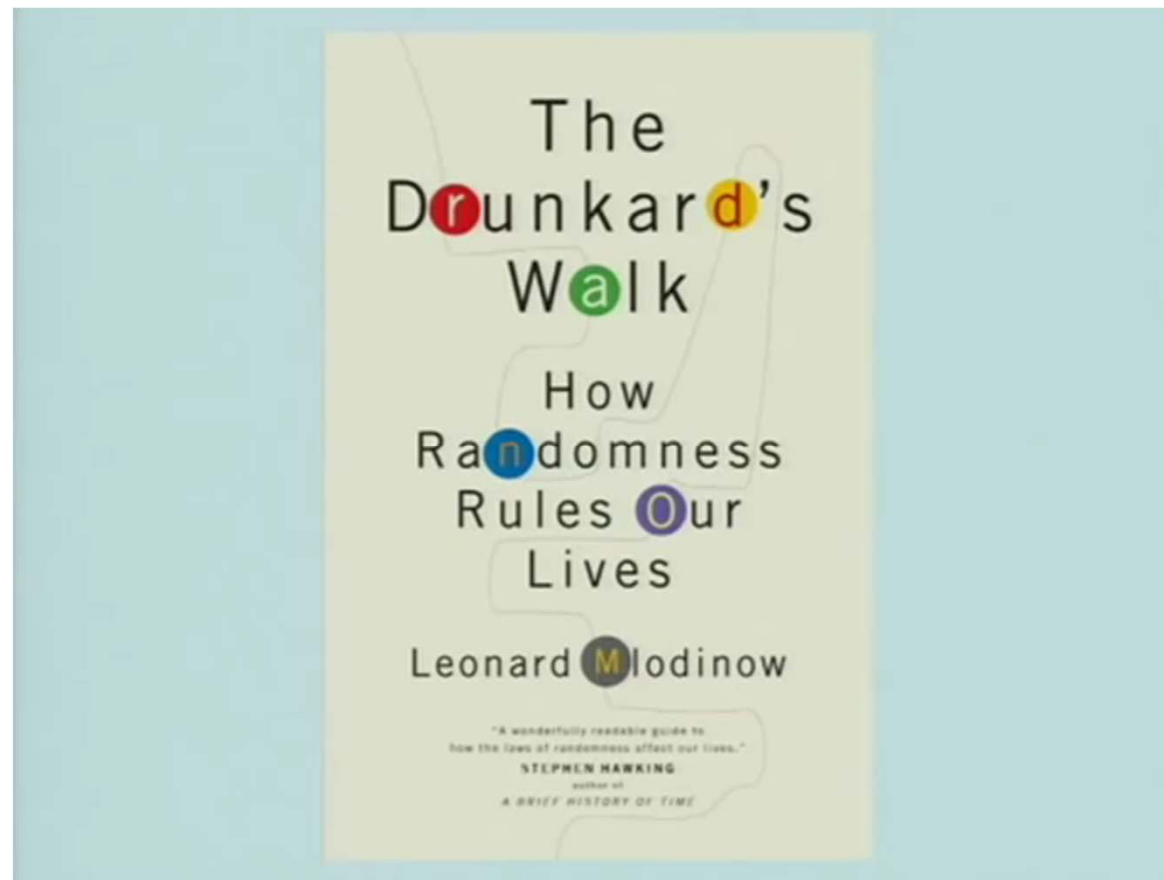
- Euclid's Window: the Story of Geometry from Parallel Lines to Hyperspace
- Feynman's Rainbow: a Search for Beauty in Physics and in Life
- A Briefer History of Time
  - with Stephen Hawking
  - an international best-seller that has appeared in 25 languages.
- The Drunkard's Walk: How Randomness Rules our Lives
- Apart from books on popular science, he also has been a **screenwriter** for television series, including **Star Trek: The Next Generation** and **MacGyver**.

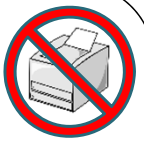




# Watch Mlodinow's talk

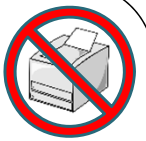
- Delivered to Google employees
- About his book (“The Drunkard's Walk”)





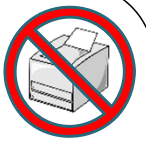
# Mlodinow's Prologue

- A few years ago a man won the Spanish national lottery with a ticket that ended in the number 48.
- Proud of his “accomplishment,” he revealed the theory that brought him the riches.
- “I dreamed of the number 7 for seven straight nights,” he said, “and 7 times 7 is 48.”
- Those of us with a better command of our multiplication tables might chuckle at the man’s error, but we all create our own view of the world and then employ it to filter and process our perceptions, extracting meaning from the ocean of data that washes over us in daily life.
- And we often make errors that, though less obvious, are just as significant as his.



# Examples

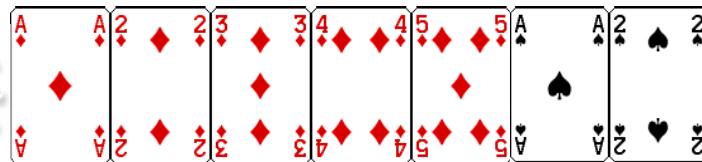
Prelude to the Theory of Probability



# Game 1:

## Seven Card Hustle





# The Seven Card Hustle

- Take **five red** cards and **two black** cards from a pack.
- Ask your friend to shuffle them and then, without looking at the faces, lay them out in a row.

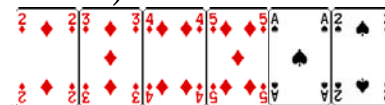


- Bet that they can't turn over three red cards.
- Explain how the bet is in their favor.

- The first draw is 5 to 2 (five red cards and two black cards) in their favor.



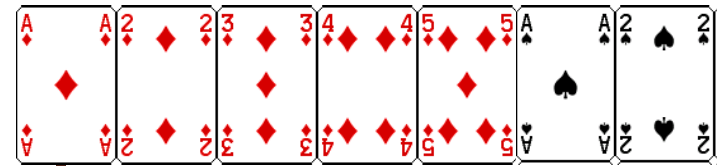
- The second draw is 4 to 2 (or 2 to 1 if you like) because there will be four red cards and two black cards left.



- The last draw is still in their favor by 3 to 2 (three reds and two blacks).

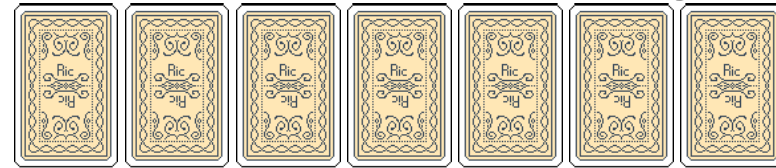


- The game seems heavily in their favor, but **YOU**, are willing to offer them **even money** that they can't do it!



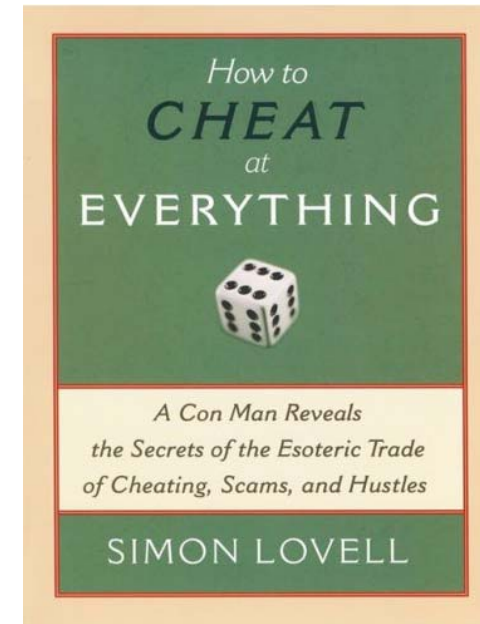
# The Seven Card Hustle

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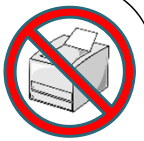


- Bet that they can't turn over three red cards.
- Explain how the bet is in their favor.
- The game seems heavily in their favor, but **YOU**, are willing to offer them **even money** that they can't do it!

Even odds or even money means 1-to-1 odds.



[Lovell, 2006]



# The Seven Card Hustle: Sol

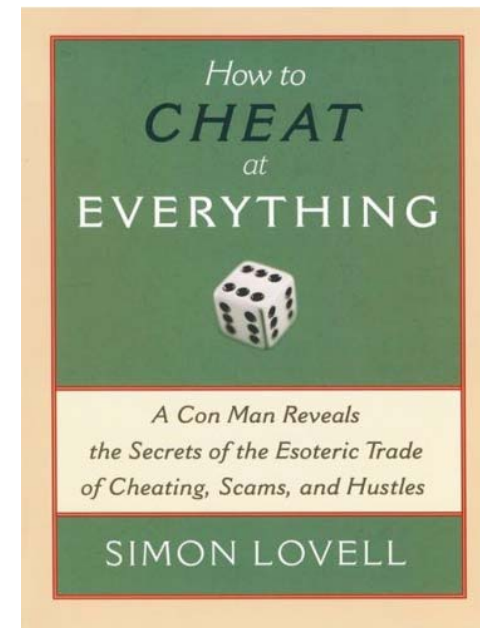
The correct probability that they can do it is

$$\frac{\cancel{5} \times \cancel{4} \times 3}{7 \times \cancel{6} \times \cancel{5}} = \frac{2}{7}$$

Do not worry too much about the math here. Some of you may be able to calculate the probability using knowledge from your high school years. We will review all of this later.

Alternatively,

$$\frac{\binom{5}{3}}{\binom{7}{3}} = \frac{5!}{\cancel{3!}2!} \times \frac{\cancel{3!}4!}{7!}$$
$$= 5 \times 4 \times 3 \times \frac{1}{7 \times 6 \times 5}$$
$$= \frac{2}{7}$$

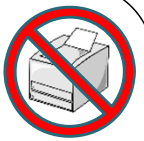


[Lovell, 2006]



# Game 2:

## Monty Hall Problem



# Monty Hall Problem (MHP): Origin

- Problem, paradox, illusion
- Loosely based on the American television game show *Let's Make a Deal*. (Thai CH7 version: ประตูดวง.)
- The name comes from the show's original host, **Monty Hall**.
- One of the most interesting mathematical brain teasers of recent times.

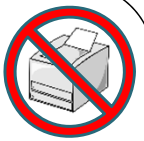


## the monty hall problem

THE REMARKABLE STORY OF MATH'S MOST  
CONTENTIOUS BRAIN TEASER

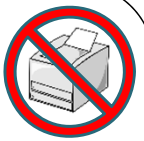


JASON ROSENHOUSE



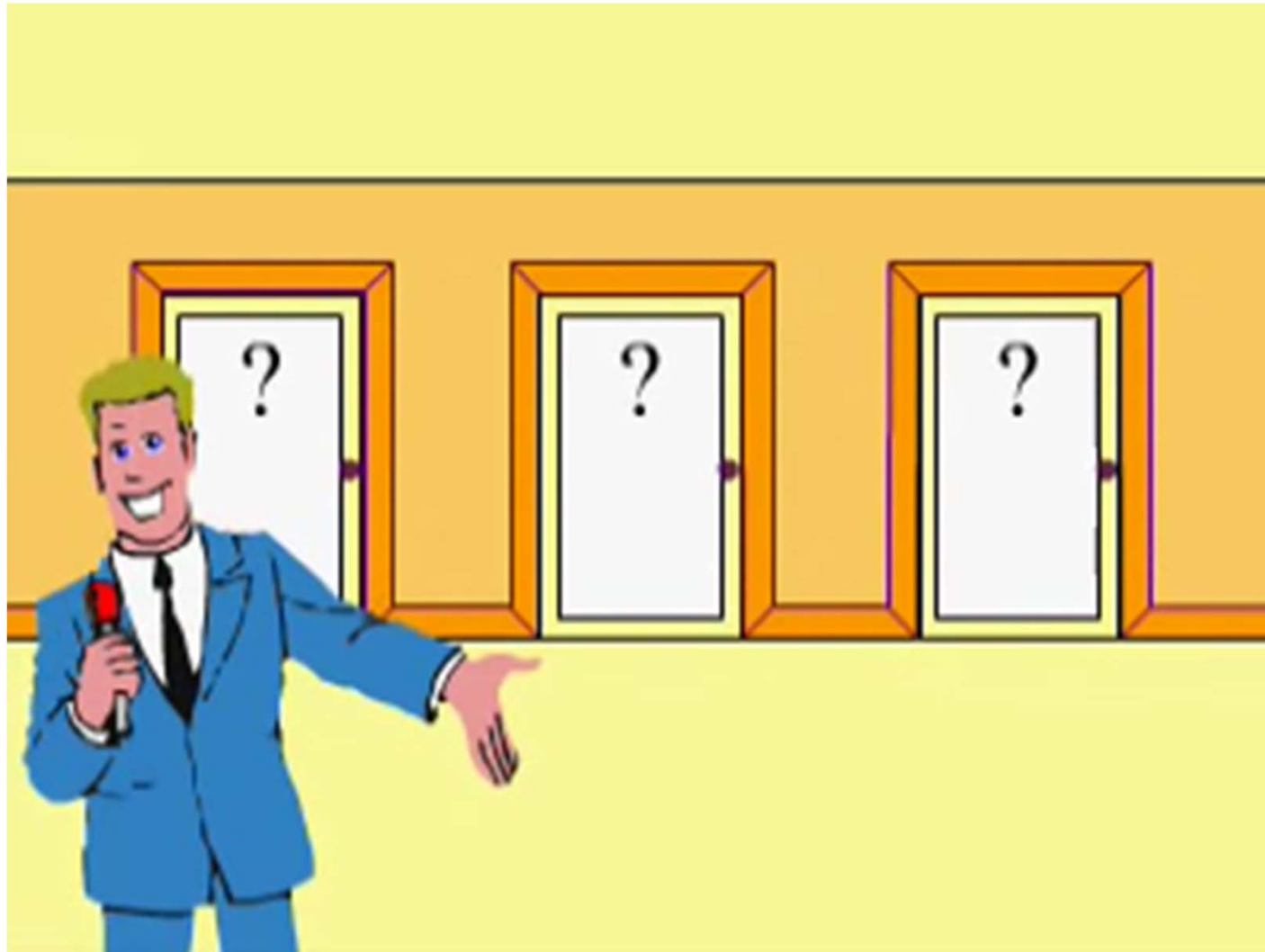
# Monty Hall Problem (MHP): Origin

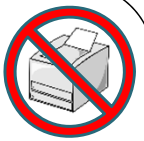




# Monty Hall Problem: Math Version

Animation with narration by Ron Clarke



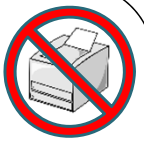


# Monty Hall Problem: Math Version

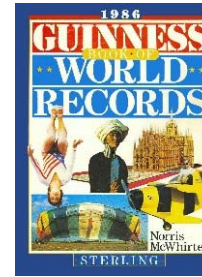
- Originally posed in a letter by Steve Selvin to the American Statistician in 1975.
- A well-known statement of the problem was published in **Marilyn vos Savant's** “*Ask Marilyn*” column in *Parade* magazine in 1990:



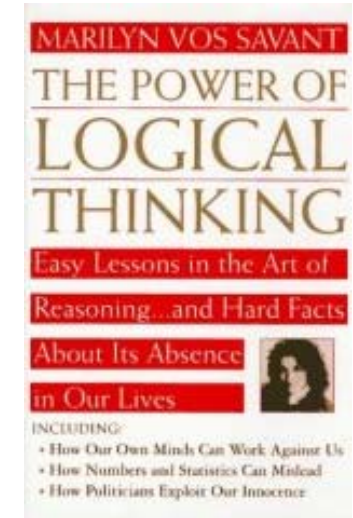
“Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, “Do you want to pick door No. 2?” Is it to your advantage to switch your choice?”

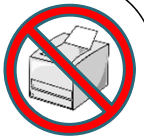


# Marilyn vos Savant



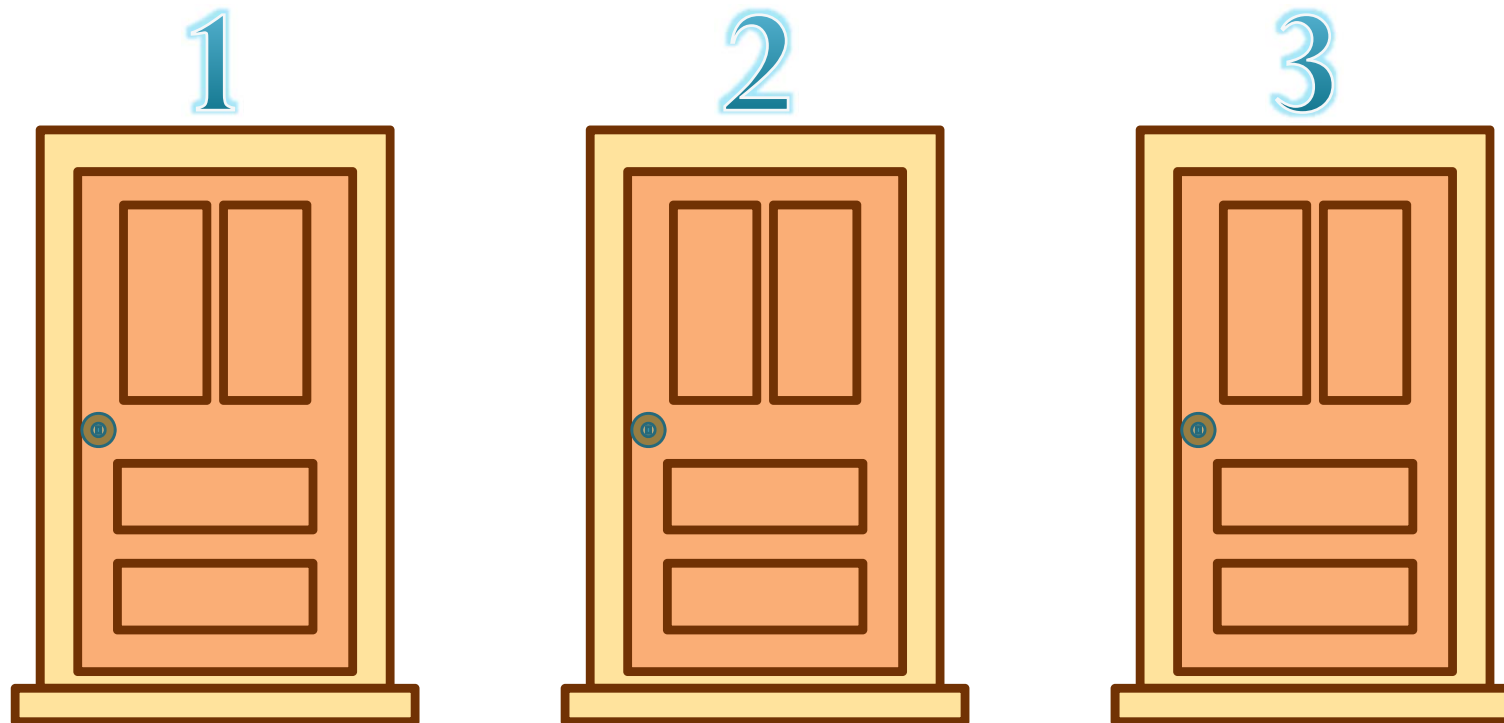
- Vos Savant was listed in each edition of the **Guinness Book** of World Records from 1986 to 1989 as having the “**Highest IQ.**”
- Since 1986 she has written “Ask Marilyn”
  - Sunday column in Parade magazine
  - Solve puzzles and answer questions from readers

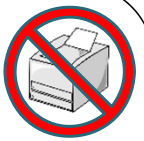




# MHP: Step 0

- There are three closed doors.
- They look identical.

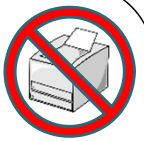




# MHP: Step 0

- Behind one of the doors is the star prize - a car.
  - The car is initially equally likely to be behind each door.
- Behind each of the other two doors is just a goat.





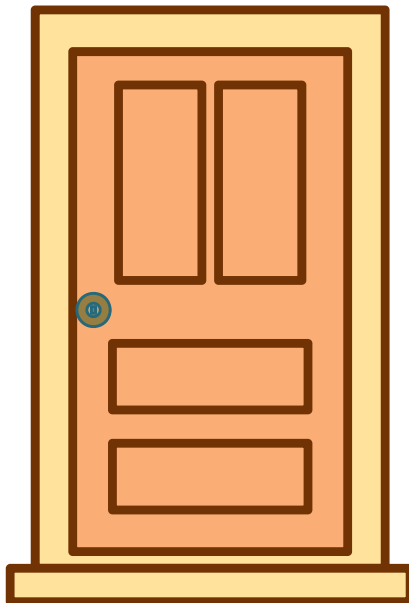
# MHP: Step 1

- Obviously we want to win the car, but do not know which door conceals the car.
- We are asked to choose a door.
- That door remains closed for the time being.

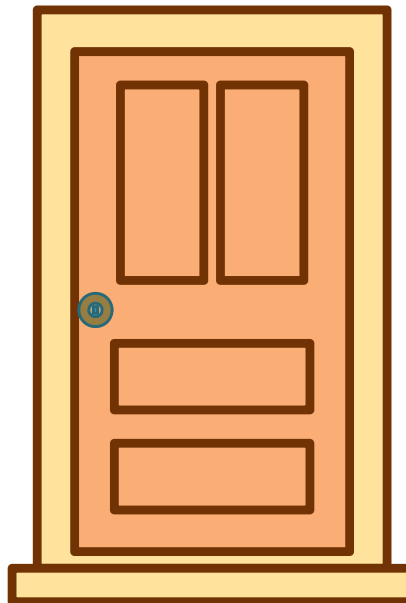
“Pick one of these doors”



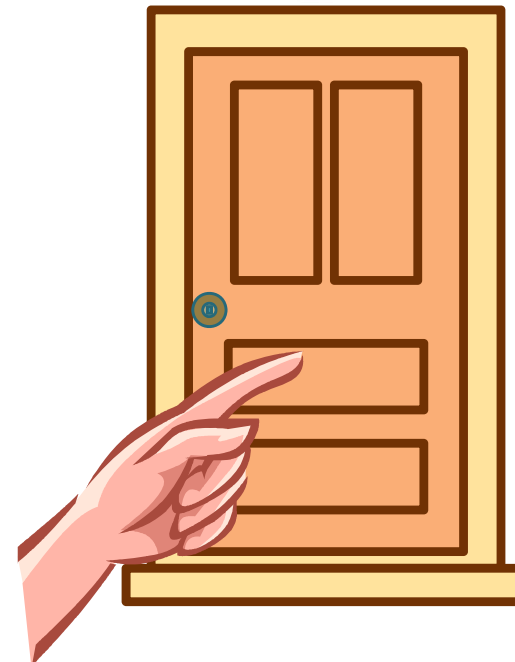
1

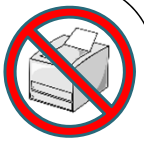


2



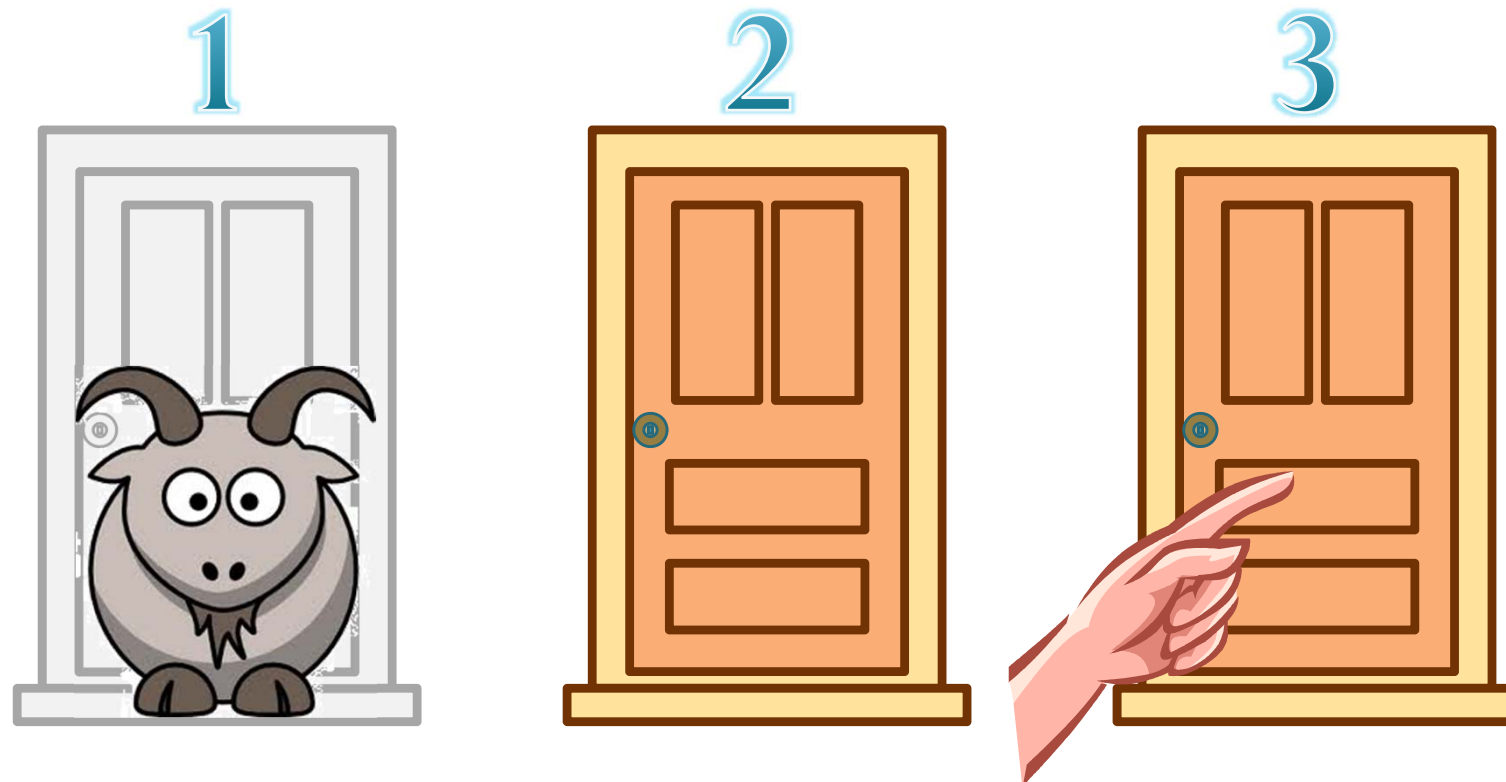
3





# MHP: Step 2

- The host of the show (Monty Hall), who knows what is behind the doors, now opens a door different from our initial choice.
  - He carefully picks the door that conceals a goat.
  - We stipulate that if Monty has a choice of doors to open, then he chooses randomly from among his options.

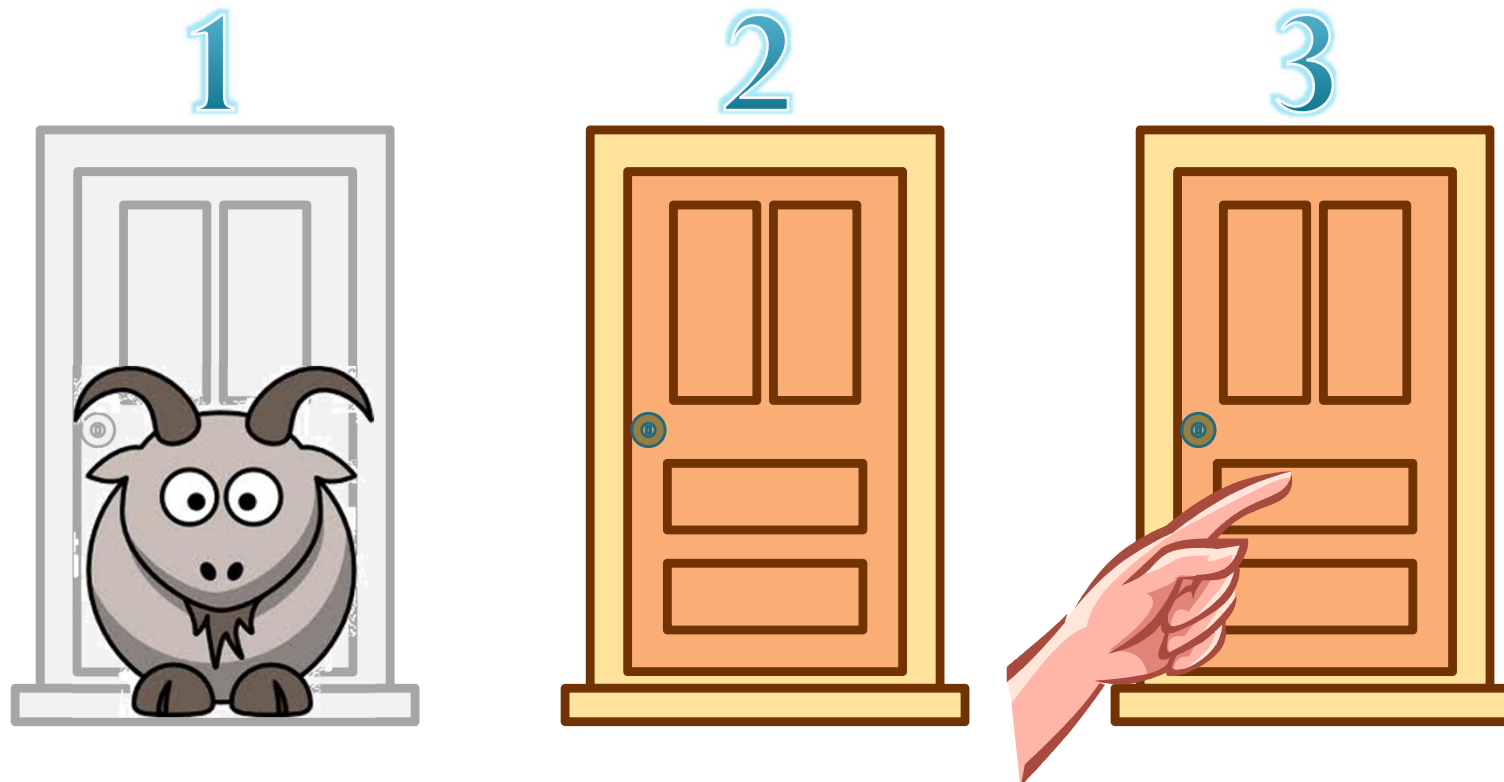


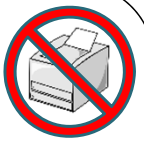
# MHP: Step 3

- Monty now gives us the options of either
  1. sticking with our original choice or
  2. switching to the one other unopened door.
- After making our decision, we win whatever is behind our door.



“Do you want to switch doors?”

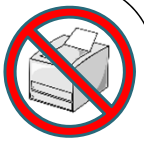




# Monty Hall Problem

Assuming that our goal is to **maximize** our **chances** of **win**ning the car, what decision should we make?

- Will you do better by **stick**ing with your first choice, or by **switch**ing to the other remaining door?
- Make no difference?



# Interactive Monty Hall

- <http://montyhallgame.shawnolson.net/>
- <http://www.shodor.org/interactivate/activities/SimpleMontyHall/>
- <http://www.math.uah.edu/stat/applets/MontyHallGame.xhtml>
- <http://scratch.mit.edu/projects/nadja/484178>
- <http://www.math.ucsd.edu/~crypto/Monty/monty.html>

**Monty Hall Game**  
This is a JavaScript-driven version of the Monty Hall Game statistics. It's an ideal challenge for gamblers before they hi

New Game Do you want to keep your previous choi

Let's do it again! Reset S

Wins Losses

Games stayed: 0  
Games stayed and won: 0

Games switched: 0  
Games switched and won: 0

© Shodor

**The Monty Hall Game**

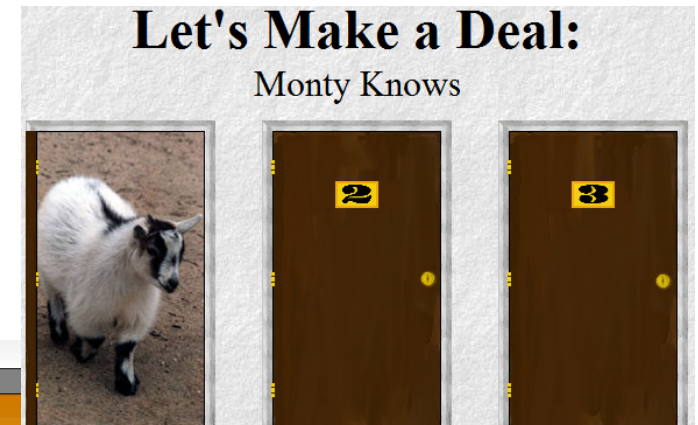
Standard

You lose! Click play to start a game.

1 Player 2 3 Host

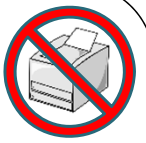
Game	G	S	W	W	Data
1	1	1	1	0	1
2	1	1	1	1	2
3	1	1	0		

0.667

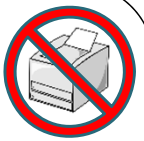


Door 1 Door 2 Door 3

© Shodor

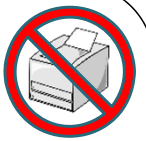


Let's *play*!

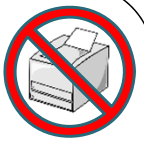


# Interactive Monty Hall





Back to the boring  
administrative stuff!



# Calendar

M	T	W	R	F
12-Aug-19	13-Aug-19	14-Aug-19	15-Aug-19	16-Aug-19
19-Aug-19	20-Aug-19	21-Aug-19	22-Aug-19	23-Aug-19
26-Aug-19	27-Aug-19	28-Aug-19	29-Aug-19	30-Aug-19
2-Sep-19	3-Sep-19	4-Sep-19	5-Sep-19	6-Sep-19
9-Sep-19	10-Sep-19	11-Sep-19	12-Sep-19	13-Sep-19
16-Sep-19	17-Sep-19	18-Sep-19	19-Sep-19	20-Sep-19
23-Sep-19	24-Sep-19	25-Sep-19	26-Sep-19	27-Sep-19
30-Sep-19	1-Oct-19	2-Oct-19	3-Oct-19	4-Oct-19
7-Oct-19	8-Oct-19	9-Oct-19	10-Oct-19	11-Oct-19
14-Oct-19	15-Oct-19	16-Oct-19	17-Oct-19	18-Oct-19
21-Oct-19	22-Oct-19	23-Oct-19	24-Oct-19	25-Oct-19
28-Oct-19	29-Oct-19	30-Oct-19	31-Oct-19	1-Nov-19
4-Nov-19	5-Nov-19	6-Nov-19	7-Nov-19	8-Nov-19
11-Nov-19	12-Nov-19	13-Nov-19	14-Nov-19	15-Nov-19
18-Nov-19	19-Nov-19	20-Nov-19	21-Nov-19	22-Nov-19
25-Nov-19	26-Nov-19	27-Nov-19	28-Nov-19	29-Nov-19
2-Dec-19	3-Dec-19	4-Dec-19	5-Dec-19	6-Dec-19
9-Dec-19	10-Dec-19	11-Dec-19	12-Dec-19	13-Dec-19

Lectures

Exams

Please Double-Check Exam Dates!



# Calendar (Google)

Available on the course website.

Today ◀ ▶ August 2019 Week Month Agenda

Sun	Mon	Tue	Wed	Thu	Fri	Sat
28 10:00 Office Ho	29 10:00 Office Ho	30 15:00 Office Ho	31 15:00 Office Ho	Aug 1 15:00 Office Ho	2 15:00 Office Ho	3
4 10:00 Office Ho	5 10:00 Office Ho	6 15:00 Office Ho	7 15:00 Office Ho	8 15:00 Office Ho	9 15:00 Office Ho	10
11 National Moth	12 Classes begin	13 10:40 ECS315 L	14 15:00 Office Ho	15 10:40 ECS315 L	16 13:00 ECS315/:	17
18 10:00 Office Ho	19 10:40 ECS315 L	20 15:00 Office Ho	21 15:00 Office Ho	22 10:40 ECS315 L	23 13:00 ECS315/:	24
25 10:00 Office Ho	26 Last day to ac	27 First day of w	28 15:00 Office Ho	29 10:40 ECS315 L	30 13:00 ECS315/:	31

Events shown in time zone: Indochina Time - Bangkok

### ECS315: Probability and Random Processes

Everything we do, everything that happens around us, obeys the laws of probability. We can no more escape them than we can escape gravity... "Probability," a philosopher (Bishop Butler) once said, "is the very guide of life." We are all gamblers who go through life making countless bets on the outcome of countless actions.

Every field of science is concerned with estimating probability. A physicist calculates the probable path of a particle. A geneticist calculates the chances that a couple will have blue-eyed children. Insurance companies, businessmen, stockbrokers, sociologists, politicians, military experts - all have to be skilled in calculating the probability of the events with which they are concerned.

[Gardner, 1986]

**Synopsis**

Probability theory is the branch of mathematics that tells us how to estimate degrees of probability. If an event is certain to happen, it is given a probability of 1. If it is certain not to happen, it has a probability of 0.

This course introduces the principles of probability and random processes to undergraduate students in electronics and communication. The topics to be covered include random experiments, events, probability, discrete and continuous random variables, probability density function, cumulative distribution function, functions of random variables, expectations, law of large numbers, central limit theorem, introduction to random processes, Gaussian random process, autocorrelation and power spectral density.

**Announcements**

- Note that we also share the tutorial/make-up session with ECS332. See Google calendar below.
- This site can be accessed via [ecs315@prapun.com](mailto:ecs315@prapun.com).
- Welcome to ECS315! Feel free to look around this site.

**General Information**

- Instructor:** Asst. Prof. Dr.Prapun Suksompong ([prapun@sit.tu.ac.th](mailto:prapun@sit.tu.ac.th))
- Office:** BKD, 6th floor of Sirindhiralai Building
- Office Hours:** See Google calendar below.
- Lectures:** See Google calendar below.

**Problem Set**

a. HW 1 (Due: ...)

b. ...

Events shown in time zone: Indochina Time - Bangkok

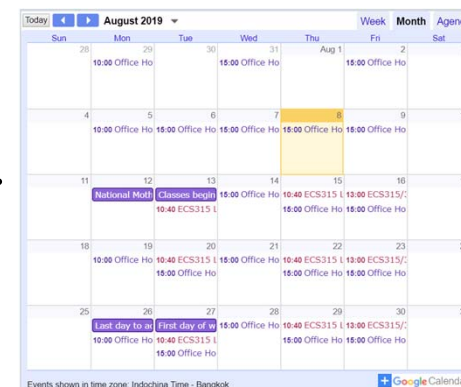
**Reading Assignment**

a. Section 1.2 in the lecture notes

b. Section 2.5 in the lecture notes

# Help and Office Hours

- Get some help!
  - Do not wait until the final exam time or after the grade is out.
  - Right after lecture is always a good time to ask question.
- Office Hours
  - Tentative Time: T,F 14:30-15:30
  - Check Google Calendar on the course website.
  - Appointment can be made.
  - Feel free to come to my office and chat!
  - Don't be shy.



Asst.Prof.Dr.Prapun Suksompong - 1/2019					
	9.00-10.20	10.40-12.00	13.00-14.20	14.40-16.00	16:00-17:00
MON	OH		JAE	MEETING	
TUE		ECS315 BKD 3510		ECS315/332 Office Hour	
WED		ECS332 BKD 2602			OH
THU		ECS315 BKD 3506			OH
FRI		ECS332 BKD 3511	ECS315/332 BKD 3511	ECS315/332 Office Hour	

## Office Hours:

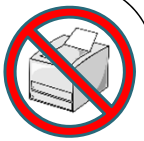
BKD, 6th floor of Sirindhralai building

**Tuesday**

**14:30-15:30**

**Friday**

**14:30-15:30**



# Grading System

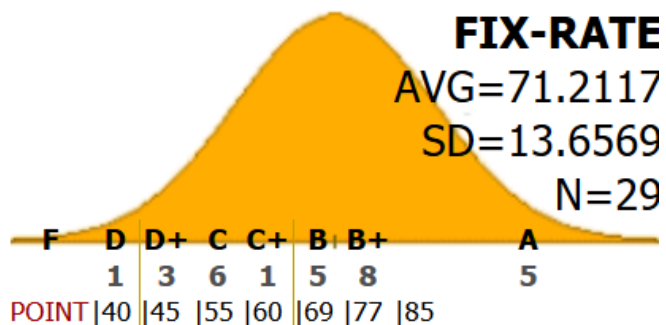
- Coursework will be weighted as follows:

Assignments	5%
Class Discussion	5%
In-Class Exercises	10%
Midterm Examination	35%
Final Examination (comprehensive)	45%

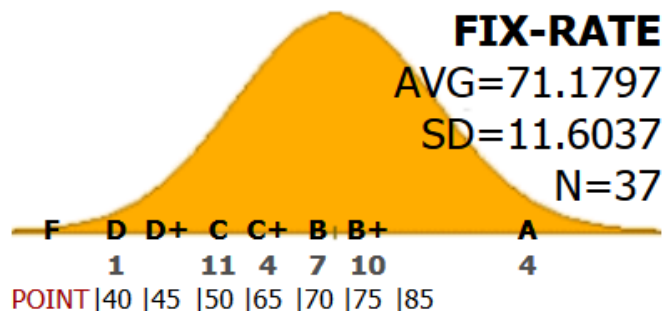
- Late HW submission will be rejected.



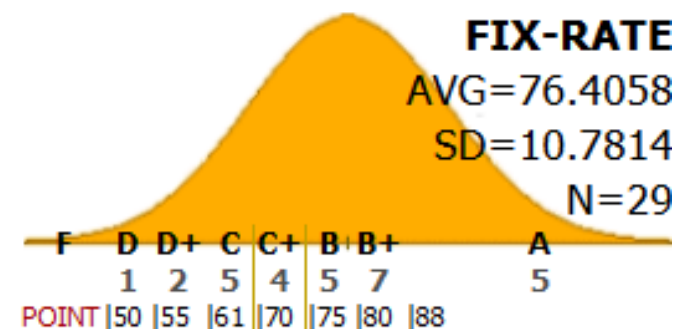
# Grading System



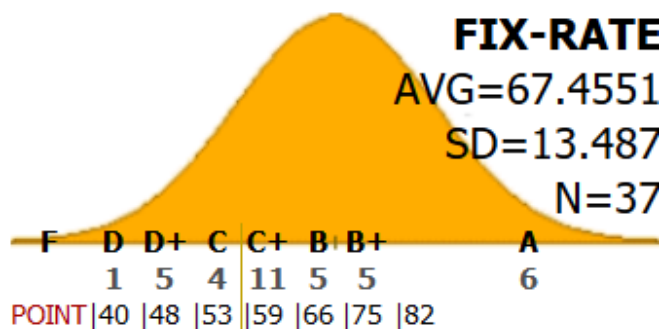
2013: CLASS GPA: 2.86



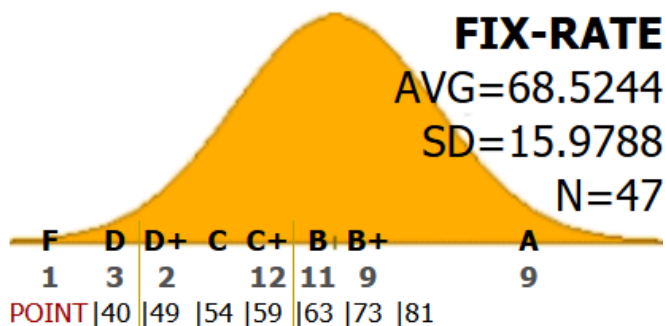
2014: CLASS GPA: 2.84



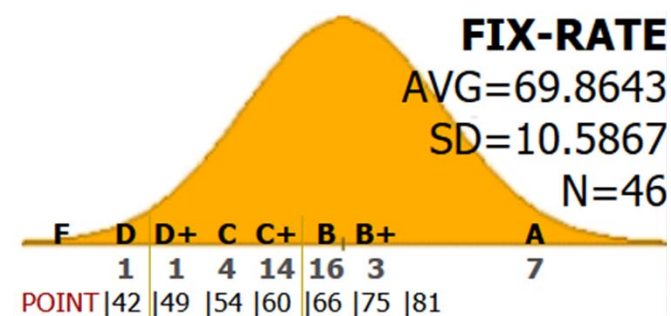
2015: CLASS GPA: 2.88



2016: CLASS GPA: 2.72



2017: CLASS GPA: 2.90



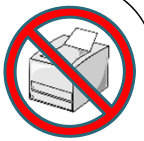
2018: CLASS GPA: 2.87

# In-Class Exercises

- Most in-class exercises will occur **without** prior warning or announcement.
  - Focus on the current topic under discussion.
- Done **in group** to **reduce pressure** and provide **opportunity**
  - for those who think they understand the course material to **explain** to their friends and see whether they really know the material under consideration
  - and
  - for those who are falling behind to get an **alternative explanation** from their peers
- Note that you **can't be in exactly the same group every time**.
  - Have to change your group members every time.
  - If you are with a friend before, then next time, form a group with someone else.

# Class Discussion

- NOT the same as class attendance!
- If you come only to **receive**, you will fall **asleep**.
  - Do not simply sit quietly in the class.
- Need **interaction** between lecturer and students.
- **Ask question** when there is something that you don't understand.
  - Don't be shy!
  - It is very likely that your friends don't understand it as well.
- If you already understand what I'm presenting, **SHOW ME!**
  - Point out the errors/typos.
  - I will raise many issues/questions in class. Try to comment on them.



# Self-Evaluation Form

- Record what you have done.
  - To be submitted right after the midterm and right after the final.

## ECS315: Self Evaluation 2019 (1)

1. The class participation score for this class is judged by how much you actively participate in the class discussion both inside and outside of the classroom.
2. Please honestly answer the following questions. Please provide as much information as possible.
3. A link is provided after submission so that you can come back and edit your own response later.

Name

Student ID

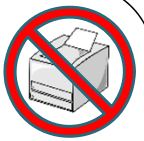
How many times have you been absent from the class? Are there any specific reason(s)? Please explain.

Note that the lowest scores among your own in-class exercise will be dropped. However, if you have valid reason for missing class on the day that we have in-class exercise(s), please indicate the date, exercise number, and the reason here. (No credit for incomplete information.) Make sure that you also submit/email supporting document/evidence to Dr.Prapun (if you haven't done so).

How many times have you participated (provided comments, asked questions, answered questions, etc) in the lectures? Be specific. Provide some short description for each event. Number alone does not count.

How many times have you correctly informed the instructors the typo or mistake on the whiteboard/slides/hw/etc? Provide short description for each of the issues.

How many times have you discussed with the instructor outside of class? (Ask questions, express concerns, etc.) Be specific. Number alone does not count.



# Self-Evaluation Form:

**How many times have you participated (provided comments, asked questions, answered questions, etc) in the lectures? Be specific. Provide some short description for each event. Number alone does not count.**

Example:

Three times.

On Aug 14, I played the Monty Hall game in class.

I choose to “switch” but did not get the car.

On Aug 21, I remind Dr.Prapun that the class time was already over.

On Sep 13, Dr.Prapun worked on an example about intersection of sets. I raised my hand and answered “ $\{3,4\}$ ” which was the correct answer.

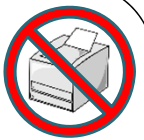
**How many times have you correctly informed the instructors the typo or mistake on the whiteboard/slides/hw/etc? Provide short description for each of the issues.**

Example:

Twice.

On Aug 16, I sent an email to Dr.Prapun about a typo in the lecture note. On page 15, the note said  $1+1 = 3$ . It should be  $1+1=2$ .

On Sep 14, In class, while we were working on page 24 of the lecture note, Dr.Prapun mistakenly wrote  $5 \times 5 = 30$ . It should be  $5 \times 6 = 30$ . I corrected him in class.



# Self-Evaluation Form (Con't)

- If you have legitimate reason for missing class on the day that we have exercise, please indicate the date, exercise number, and the reason in the self-evaluation form.
- Make sure that you also submit/email supporting document/evidence to Dr.Prapun.

**Note that the lowest scores among your own in-class exercise will be dropped. However, if you have valid reason for missing class on the day that we have in-class exercise(s), please indicate the date, exercise number, and the reason here. (No credit for incomplete information.) Make sure that you also submit/email supporting document/evidence to Dr.Prapun (if you haven't done so).**


Example:

On Oct 18, we had Exercise 5 but I have to miss the class because I broke my leg and was hospitalized for two days. I have scanned the doctor's certificate and email it to Aj.Prapun on Oct 30.



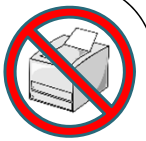
Based on the clock on my computer. (This should be approx. the same as your phone's and computer's clocks if they are synchronized properly.)

# Policy

- We will start the class **on time** and will finish **on time**.
  - I recommend arriving at least 3 minutes before the start time.
  - Raise your hand and tell me immediately if I go over the time limit.
    - Does NOT mean that I will leave the room immediately after lecture.
    - I will stay and answer questions.
- Mobile phones *must* be turned off or set in silent mode. 
- Attendance will be taken/given irregularly and randomly.
- Cheating will not be tolerated.
- Feel free to stop me when I talk too fast or too slow.

# Policy (con't)

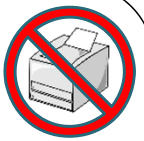
- I will surely make some **mistakes** in lectures / HW / exams.
  - Some amount of class participation scores will be reserved to reward the **first** student who informs me about each of these mistakes.
    - Grammatical errors are best informed/corrected after class.
- Unless instructed otherwise, points on exercises and exams are based on your entire solution, not your final answer.
  - You may get full credit even when you have the wrong final answer.
  - You may get **zero** even when you write down a right answer without justification.



# Policy (con't)

- Please stop me if I go over the time limit.
- Please stop me if I talk too fast.
- Please stop me if you have any question.

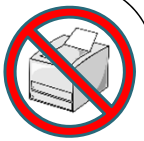




# Warning

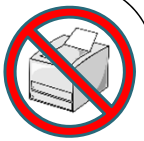
- This class is **difficult**.
  - Keep up with the lectures.
  - Make sure that you understand the concepts presented in the lecture before you go home.
- I will **evaluate** your understanding of the course **regularly** through
  - In-class exercises/activities
  - Exams





# Difficulty in ECS315

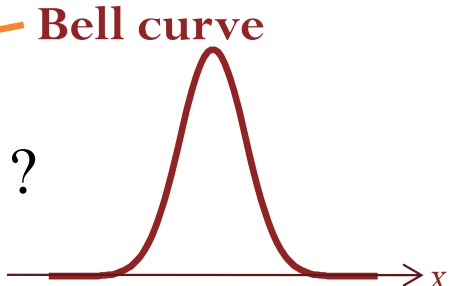
- Combinatorics (counting)
  - Solving word problems
  - Not the main focus of this class but unavoidable if you want to solve/consider interesting questions
- Calculus
  - Can be messy
- Concept of probability
  - Most students do not learn probability until two or three exposures to it.
- Large number of definitions, formulas and equations
  - No need to remember a lot of formulas if you understand them

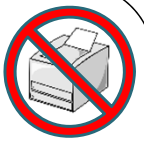


# Prerequisite

- Working knowledge of calculus
- Some MATLAB skills for doing HWs and understanding in-class demo
- Frequency domain analysis (Fourier transform)

Soon, we will need to find

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = ?$$
$$\mathcal{F} \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right\} = ?$$


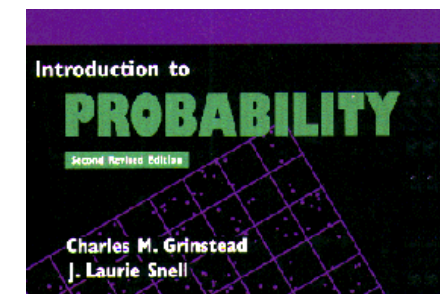
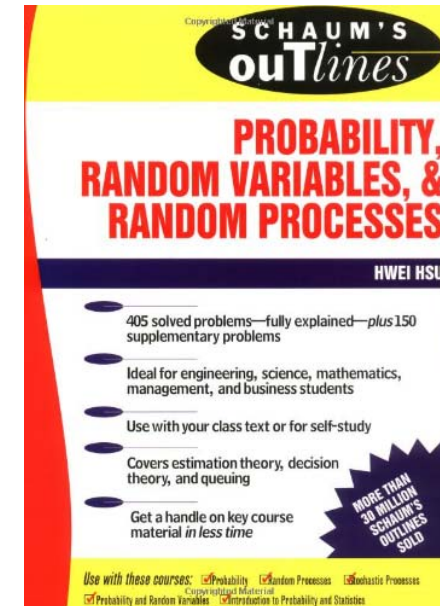


# Remarks

- Get as much **legitimate** help as you can
- **Participate actively in class** and outside of class
  - Record what you have done.
- If you feel that the class is very easy, you might overlook something.
- If you feel that the class is very difficult, you are probably not the only one who feel that way.
  - Don't give up. Chat with me.
  - It takes me a long time to feel comfortable with these materials; yet, I still make mistakes.
- My notation can be different from the textbook.
  - Every notation has some advantages and disadvantages.

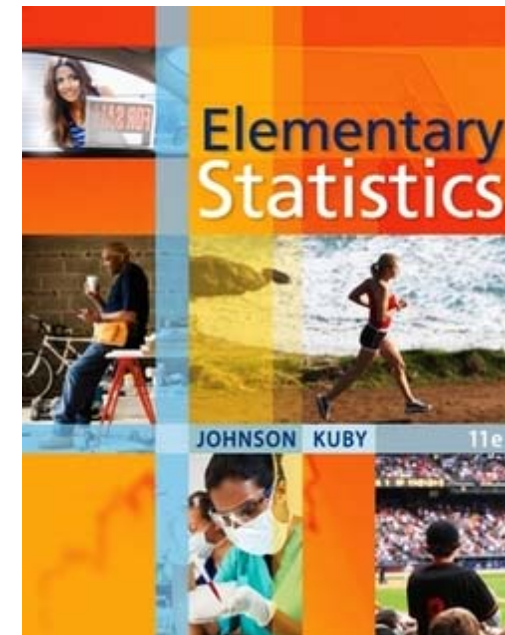
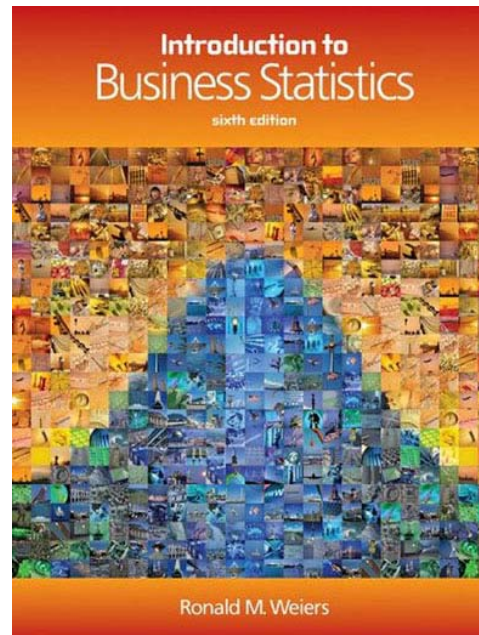
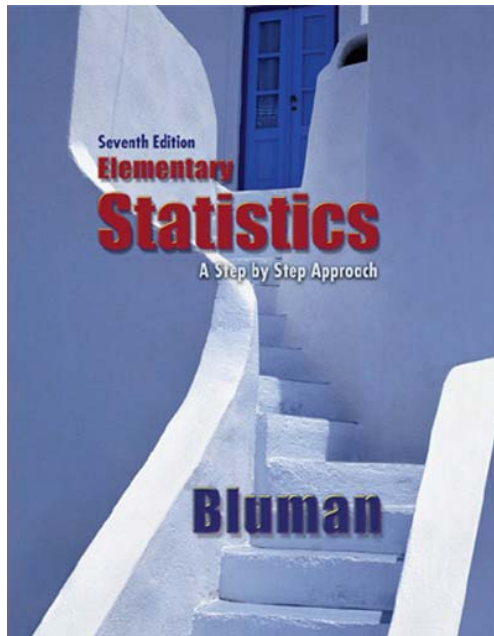
# Need More Examples or Practice?

- Textbook in the **library**: **Schaum's** outline of theory and problems of probability, random variables, and random processes / Hwei P. Hsu. Call No. QA273.25 H78 1997
- Free pdf textbook:  
Introduction to Probability by **Grinstead** and **Snell**  
[http://www.dartmouth.edu/~chance/teaching\\_aids/books\\_articles/probability\\_book/book.html](http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.html)



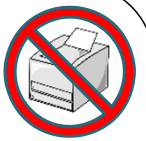
# Easier References

For those who feels that this course is difficult, here are some easier references.



More beautiful pictures. Less technical.

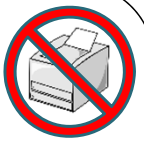
Less applicable for content after the midterm.



# Monty Hall Problem: a short revisit

Assuming that our goal is to **maximize** our **chances** of **win**ning the car, what decision should we make?

- Will you do better by **stick**ing with your first choice, or by **switch**ing to the other remaining door?
- Make no difference?

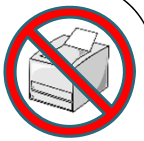


# Interactive Monty Hall

A lot of people choose to “stay”.

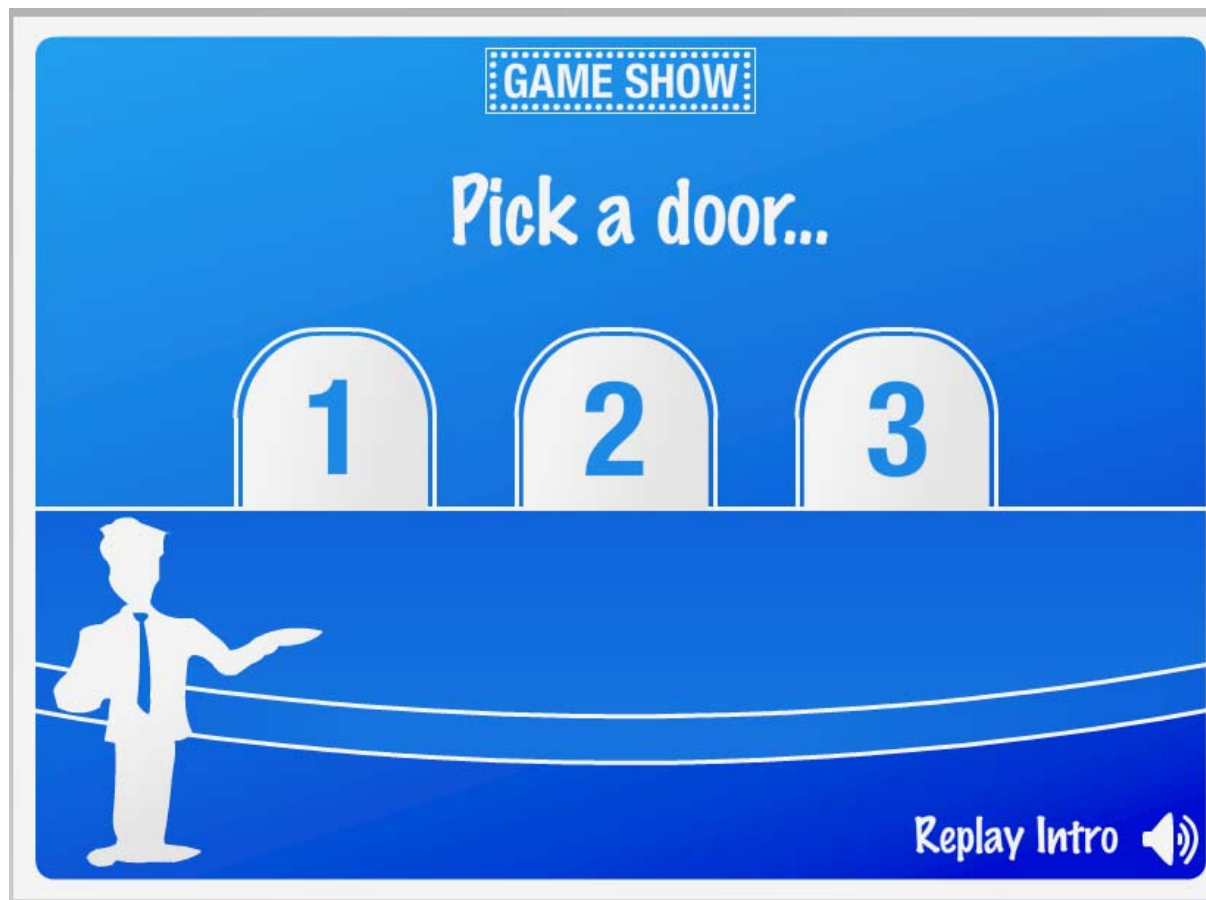
players who **STAY** have won 216085 cars out of 644093 games yielding a winning percentage of 34%

players who **SWITCH** have won 221880 cars out of 334427 games yielding a winning percentage of 66%



# Interactive Monty Hall

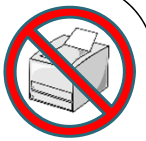
Winning probability:



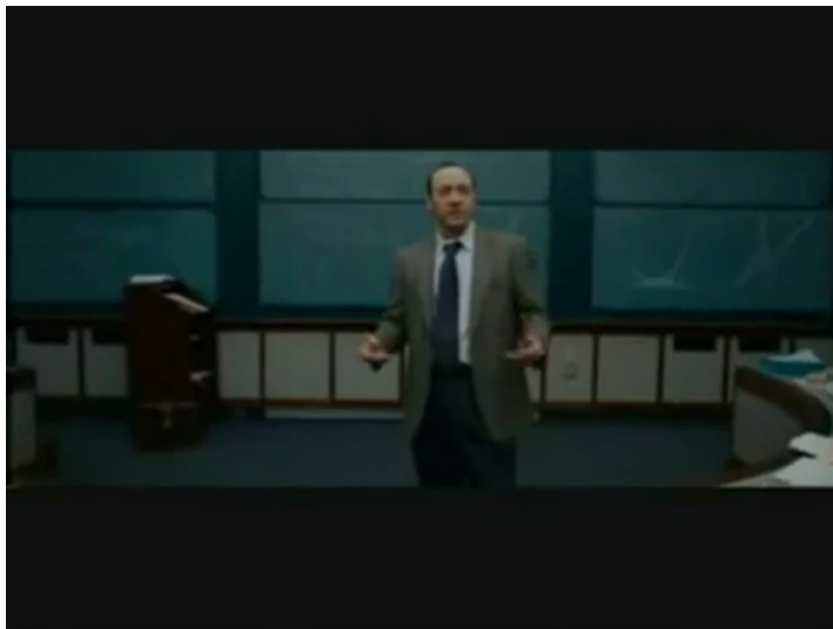
players who **STAY**  
have won 216085  
cars out of 644093  
games yielding a  
winning percentage  
of 34% **1/3**

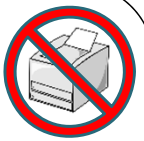
players who **SWITCH**  
have won 221880  
cars out of 334427  
games yielding a  
winning percentage  
of 66% **2/3**

# Monty Hall Problem: vos Savant's Answer



“You **double** your chances of **winning** by **switching** doors.”

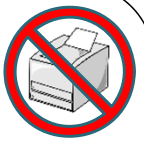




# Monty Hall Problem: Controversy

- Approximately **10,000 readers**,
  - including nearly **1,000 with PhDs**
  - (many of them **math professors**),
  - wrote to the magazine
  - **claiming** the published solution was **wrong**.
- “You blew it,” wrote a mathematician from George Mason University.
- From Dickinson State University came this: “I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.”

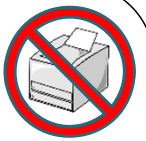
[Mlodinow, 2008, p 42-45]



## Controversy (2)

- From Georgetown: “How many irate mathematicians are needed to change your mind?”
- And someone from the U.S. Army Research Institute remarked, “If all those Ph.D.s are wrong the country would be in serious trouble.”
- When told of this, Paul **Erdős**, one of the leading mathematicians of the 20th century, said, “That's **impossible**.”
  - Then, when presented with a formal mathematical proof of the correct answer, he still didn't believe it and grew angry.
  - Only after a colleague arranged for a **computer simulation** in which Erdős watched hundreds of trials that came out 2-to-1 in favor of switching did Erdős concede that he was wrong.





Let's learn some concepts  
so that we can analyze  
interesting examples!